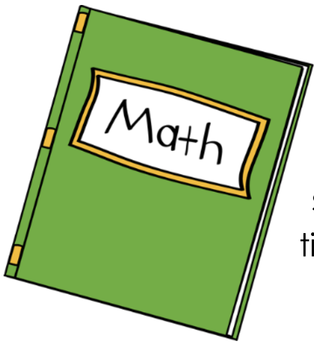
Sample Math Activity – Positive Integers Divisible by 3



Math contains a large number of "fun" but challenging activities and challenges for students. A math tutor can have a repertoire of such activities and draw an appropriate one out of the bag when time and the situation seem right. Here is an example:

Positive Integers Divisible by 3

We know that some positive integers are exactly divisible by the number 3 and others are not. The number 7,341 is an example of 4-digit number divisible by 3:

7341/3 = 2447

Now, let's form other 4-digit numbers from the four digits 7, 3, 4, and 1. Examples include 3741, 1437, 4137, and so on. It turns out that each of these is exactly divisible by 3.

3741/3 = 1247 1347/3 = 449 4137/3 = 1379

Interesting. Perhaps we have found a pattern. Try some other 4-digit numbers formed from, the digits 7, 3, 4, and 1. It turns out that each of the 4-digit numbers you form will be evenly divisible by 3. [It also works for 2-digit numbers, 3-digit numbers, et cetera.]

Here are some more questions:

1. How many different 4-digit numbers can one make from the digits 7, 3, 4, 1? This question is relevant because we may want to test every one of them to see if it is divisible by 3.

Note to tutors: Use a 3-digit version of this question for tutees you feel will be overwhelmed by the 4-digit version. Your goal is to introduce the idea of careful counting and a situation in which your tutee can experience success.

2. Are there other 4-digit numbers that are divisible by 3 and such that any number formed from these four digits is divisible by 3? This question is relevant as we work to find then the divisibility conjecture might be true. Some exploration will lead you to a conjecture that this "divisible by 3" pattern works on the variety of 4-digit numbers that

you try. Of course, that does not prove that it works for all 4-digit numbers that are divisive by 3. How many different 4-digit numbers are there that are divisible by 3? Is it feasible for a person to list all of these and then test for each one all of the 4-digit numbers that can be made from the digits? (A computer could complete this task in a small fraction of a second.)

3. Does the divisible by 3 property we have explored for 4-digit numbers also hold for 2-digit numbers, 3-digit numbers, 5-digit numbers, and so on? Some trials might well lead you to conjecture that the answer is "yes." But now, we have a situation in which an exhaustive test of all possible numbers is not possible. What is needed next is a "mathematical proof" that the conjecture is correct, or finding an example for which the conjecture is not correct.

4. Explore the following conjectures:

> 4a. If the sum of the digits in a positive integer is divisible by 3, then the integer is divisible by 3.
>
> 4b. If a positive integer is divisible by 3, then the sum of its digits is divisible by 3.

Source: *Becoming a Better Math Tutor* by David Moursund & Robert Albrecht http://i-a-e.org/downloads/free-ebooks-by-dave-moursund/208-becoming-a-better-math-tutor/file.html